SECOND PUBLIC EXAMINATION

Honour School of Physics Part C: 4 Year Course

Honour School of Physics and Philosophy Part C

C5: PHYSICS OF ATMOSPHERES AND OCEANS

TRINITY TERM 2011

Tuesday, 21 June, 9.30 am – 12.30 pm

Answer **four** *questions.*

Start the answer to each question in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do NOT turn over until told that you may do so.

With wavenumber ν measured in cm^{-1}, the radiance from the surface of a black body of temperature T (in K) is given by

$$B_\nu(T)\,\mathrm{d}\nu = \frac{c_1\nu^3\,\mathrm{d}\nu}{\mathrm{e}^{c_2\nu/T}-1}\,\mathrm{W\,m^{-2}\,sr^{-1}},$$

where $c_1 = 1.191043\times10^{-8}$ and $c_2 = 1.438775$.

The following equation of motion for a fluid relative to coordinates stationary with respect to a rotating body may be assumed:

$$\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} + 2\boldsymbol{\Omega}\times\mathbf{V} = -\frac{1}{\rho}\nabla p - g\mathbf{k}\,,$$

where $\mathbf{V}$ is the velocity of the fluid, $\boldsymbol{\Omega}$ is the angular velocity of the body, p is the pressure, ρ is the fluid density, g is the acceleration due to gravity and $\mathbf{k}$ is a unit vertical vector.

Unless otherwise stated, x and y refer to the horizontal eastward and northward directions and z the upward direction with corresponding velocities u, v and w.

$$1000\,\mathrm{mb} = 1000\,\mathrm{hPa} = 10^5\,\mathrm{Pa}.$$

1. The Equation of Radiative Transfer for upward propagation in a plane-parallel atmosphere with no thermal sources is written

$$\mu \frac{dL_\nu(t;\theta,\phi)}{dt} = L_\nu(t;\theta,\phi) - L_\nu^{\mathrm{S}}(t;\theta,\phi).$$

State the assumptions made in the plane parallel approximation for radiative transfer and describe the terms in this equation. [6]

An atmospheric layer of optical depth $t^* \ll 1$ is composed of identical particles which have a single scatter albedo of $\tilde{\omega}$. Show that the upward radiation field at the top of such a layer illuminated by the Sun alone can be approximated by

$$L_\nu(\mu,\phi) \approx E^{\mathrm{Sun}} \frac{t^*}{\mu} \frac{\tilde{\omega} P(\mu_0,\phi_0,\mu,\phi)}{4\pi} .$$

Here E^{Sun} is the solar irradiance at the top of the Earth's atmosphere and $P(\mu_0,\phi_0,\mu,\phi)$ is the particle phase function, where μ_0 and ϕ_0 are the cosine of the solar zenith angle and the solar azimuth angle respectively. Assuming that such a layer is above a black surface, calculate the reflectance of the layer for particles where $\tilde{\omega} = 1, P(\mu_0,\phi_0,\mu,\phi) = \mu$ and the Sun is overhead. [12]

A proposed solution to combat global warming is to inject particles in the stratosphere to reflect solar radiation into space. Calculations suggest a layer reflectance of 0.01 is required, assuming an overhead Sun. By modelling the Earth as a Sun-facing disc with a radius equivalent to the mean radius of the Earth, estimate the thickness of an atmospheric layer required to achieve a reflectance of 0.01. Assume the layer is composed of particles with a radius of 1 μm and the particle number density is 10 particles per cm^3. The extinction efficiency at 1 μm can be taken as 2. [7]

2. Describe the two main types of infrared detector commonly found in Earth-observing space based instruments and the typical sources of noise associated with them. Which noise source is typically minimised by chopping the signal? [6]

The signal-to-noise ratio for an infrared instrument can be written as

$$S = LTA\Omega\Delta\nu D^* \frac{\sqrt{\Delta t}}{\sqrt{a}} ,$$

where D^* is the detectivity of the infrared detector (in $\mathrm{cm\,Hz^{1/2}\,W^{-1}}$), $\Delta\nu$ is the detector band pass, L is the radiance incident on the detector, T is the throughput for the radiation reaching the instrument from the Earth's surface, Δt is the integration time and a is the area of the detector element in $\mathrm{cm^2}$. What is the significance in an ideal imaging system of the product $A\Omega$?

It is proposed to add a simple imaging radiometer to a small remote sensing satellite, suitable for use as part of a disaster monitoring constellation. It is expected that the satellite will be launched into a 680 km altitude Sun-synchronous orbit. What is meant by the term 'Sun-synchronous' and what are the main advantages and disadvantages of such an orbit?

The instrument is designed to have a nadir, ground track resolution of 1 km diameter and has an infrared filter of spectral width 10 $\mathrm{cm^{-1}}$ centred at 1205 $\mathrm{cm^{-1}}$ for monitoring the surface temperature of the Earth. The f-number (the ratio of the focal length to the lens diameter) of the final optical element subtended by the 0.5 $\mathrm{mm^2}$ detector is 2. If the optical throughput of the infrared filter, instrument optics and chopper is 75% and the transmission from the surface to space by the atmosphere is 90%, estimate the diameter of the entrance aperture of the instrument. [8]

Estimate the signal-to-noise ratio for the instrument, assuming the detector has a D^* of $2 \times 10^8\,\mathrm{cm\,Hz^{1/2}\,W^{-1}}$ when viewing the surface of the Earth at a temperature of 288 K. What is the minimum detectable temperature difference the radiometer can measure at 288 K? How would the calibration of the instrument be maintained whilst in orbit? [11]

[The mass of the Earth is 6×10^{24} kg.]

3. Assuming that blackbody emission can be expressed as a linear function of optical depth, show that radiance received at the top of the atmosphere by a nadir pointing radiometer has been emitted from an optical depth t of approximately one. Use this result to explain the concepts of limb darkening and limb brightening. [4]

Consider a planetary atmosphere above a surface at temperature T_s whose emissivity is unity. A radiometer observes this planet from space at angles of 0°, 60° and 70.53° to the nadir before viewing a stable blackbody calibration target whose temperature is kept constant at 274 K. The radiance measured when observing at 0°, 60° and 70.53° is divided by the radiance measured off the blackbody and recorded as R_0, R_{60} and R_{70}. The atmosphere of the planet can be modelled as two non-scattering layers with the top layer at temperature T_1 and the bottom layer at temperature T_2. If the layers have the same optical depth $t = \ln 2$ show that the surface and layer temperatures can be estimated using

$$\begin{bmatrix} B(T_s)/B(274) \\ B(T_2)/B(274) \\ B(T_1)/B(274) \end{bmatrix} = \begin{bmatrix} 7 & -14 & 8 \\ -\frac{11}{3} & 18 & -\frac{40}{3} \\ \frac{1}{3} & -2 & \frac{8}{3} \end{bmatrix} \begin{bmatrix} R_0 \\ R_{60} \\ R_{70} \end{bmatrix}$$

where $B(T)$ is the radiance measured by the radiometer when observing a blackbody at temperature T. [12]

Given that the radiometer measures at 800 cm^{-1}, what is the surface temperature T_s, given that R_0, R_{60} and R_{70} have values of 1, 0.96 and 0.97 respectively? In practice such a simple inversion of radiance into temperature does not work: why not? [9]

[The Planck function at 800 cm^{-1} or its inverse can be found by linear interpolation of the table below:

Temperature / K	Planck Function / W m^{-2} sr^{-1} cm^{-1}
274	0.0927
284	0.108
294	0.124
304	0.141

]

4. The equations of motion for linearised disturbances about a resting, uniformly stratified, uniformly rotating, incompressible fluid are:

$$
\begin{aligned}
\frac{\partial u}{\partial t} - fv + \frac{1}{\rho_0}\frac{\partial p}{\partial x} &= 0 \, , \\
\frac{\partial v}{\partial t} + fu + \frac{1}{\rho_0}\frac{\partial p}{\partial y} &= 0 \, , \\
\frac{\partial w}{\partial t} - b + \frac{1}{\rho_0}\frac{\partial p}{\partial z} &= 0 \, , \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \, , \\
\frac{\partial b}{\partial t} + N^2 w &= 0 \, ,
\end{aligned}
$$

where $b = -g\,(\rho - \rho_0)/\rho_0$ is buoyancy and the remaining symbols have their usual meanings. The fluid lies between solid boundaries at $z = 0$, H.

Derive the internal wave equation,

$$
\frac{\partial^2}{\partial t^2}\left(\nabla_h^2 w + \frac{\partial^2 w}{\partial z^2}\right) + N^2 \nabla_h^2 w + f^2 \frac{\partial^2 w}{\partial z^2} = 0 \, ,
$$

where ∇_h is the horizontal gradient operator. [7]

Consider a separable trial solution of the form $w = \mathrm{e}^{-\mathrm{i}\omega t}\,\tilde{w}(x, y)\,G(z)$. Deduce the corresponding forms for u, v, p and b.

Derive the solution for $G(z)$. Show that a solution for $\tilde{w}(x, y)$ is a plane wave satisfying the dispersion relation

$$
\omega^2 = \frac{c^2(k^2 + l^2) + f^2}{1 + c^2(k^2 + l^2)/N^2} \, ,
$$

stating the admissible values for c. [11]

Obtain and physically interpret the short wave and long wave limits of this dispersion relation.

Calculate the period of short and long waves at 30°N in the ocean, given a uniform stratification of $-\partial \rho / \partial z = 10^{-3}\,\mathrm{kg\,m^{-4}}$. [7]

5. Suppose that a localised downwelling source of abyssal water in the high-latitude North Atlantic, of magnitude $S = 20\,\text{Sverdrups}$ (where $1\,\text{Sverdrup} = 10^6\,\text{m}^3\,\text{s}^{-1}$), is balanced by uniform upwelling, w^*, over the remainder of the global ocean. Estimate w^*. [3]

The abyssal ocean is represented in a simple model as a homogeneous slab of uniform thickness, with prescribed uniform upwelling w^* through its upper boundary. The equations of motion are:

$$\begin{aligned} -fv + \frac{1}{\rho_0}\frac{\partial p}{\partial x} &= 0 \ , \\ fu + \frac{1}{\rho_0}\frac{\partial p}{\partial y} &= -rv \ , \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \ . \end{aligned}$$

Briefly discuss any further approximations that have been made in obtaining these equations, and the extent to which these approximations are justified.

By deriving a vorticity equation, or otherwise, show that the depth-integrated meridional velocity consists of a basin-wide component, fw^*/β, and a western boundary component.

Deduce an expression for the width of the western boundary current. [9]

Show that the *southward* volume transport of the western boundary current in the Atlantic is approximately

$$S\left(1 - \frac{\Delta A}{A}\right) + \frac{fLw^*}{\beta} \ ,$$

where ΔA is the surface area of the Atlantic north of the chosen latitude, A is the surface area of the global ocean, and L is the width of the Atlantic at the chosen latitude.

Write down a similar expression for the *northward* volume transport of the western boundary current in the Pacific.

Estimate the transport of the western boundary current at 45°N in the Atlantic. [9]

Sketch the global abyssal circulation predicted by this model. [4]

6. Discuss briefly why the geostrophic balance assumption is important in the initialization of numerical weather forecasts. Show that, if the flow is geostrophic,

$$f\frac{\partial u}{\partial \ln p} = R\left(\frac{\partial T}{\partial y}\right)_p ,$$

$$f\frac{\partial v}{\partial \ln p} = -R\left(\frac{\partial T}{\partial x}\right)_p ,$$

where p is pressure, u and v are the zonal and northward velocity components of the thermal wind and f is the Coriolis parameter. [5]

The *bearing* of the geostrophic wind is defined as the angle ϕ, measured clockwise from North, to the direction from which the wind blows. Obtain an expression relating the turning of the geostrophic wind with pressure ($\partial\phi/\partial \ln p$) to the horizontal advection of temperature $(\mathbf{u}\cdot\nabla T)_p$ on an isobaric surface. Hence show that *veering* (in which ϕ increases with height) is associated with warm advection, and *backing* (in which ϕ decreases with height) with cold advection. [6]

A radiosonde measures the wind speed between 1000 hPa and 500 hPa to remain constant with height at $15\,\mathrm{m\,s^{-1}}$. The wind is observed to be northerly between 1000 hPa and 700 hPa but then backs in direction from $\phi = 0°$ at 700 hPa to a bearing of $\phi = 300°$ at 500 hPa. Estimate the mean rate of change of temperature in $\mathrm{K\,day^{-1}}$ within the layer of the atmosphere between 1000 hPa and 500 hPa over the site of the radiosonde station, clearly stating any assumptions or approximations you make. [10]

Deduce what you can about the synoptic situation implied by these measurements, and possible significant weather to be expected during the following few hours. [4]

7. The free-surface equations for a shallow atmosphere of equivalent depth H close to the equator may be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \beta y v = -g\frac{\partial \eta}{\partial x} ,$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \beta y u = -g\frac{\partial \eta}{\partial y} ,$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 ,$$

where g is the acceleration due to gravity and η is the free surface displacement. Starting from the equation of motion in the rubric on page 1, outline how these equations are derived, stating clearly the key assumptions and approximations. [6]

Show that there exists an equatorially trapped Kelvin wave solution to the linearised form of these equations for which v is identically zero and η takes the form

$$\eta = A\exp(-y^2/L_D^2)F(x - ct) ,$$

where F is an arbitrary function and A is a constant. Find expressions for L_D and c in terms of g, H and β, and comment on the direction of phase propagation. [6]

For the planet Mars, the zonal wavenumber $m = 1$ equatorial Kelvin wave is observed to have a period of approximately one Martian day (of duration $\simeq 24$ hours). Hence estimate the equivalent depth, H, for this mode and the latitude at which its amplitude falls by a factor 1/e of its value at the equator, given $g = 3.72\,\mathrm{m\,s^{-2}}$ and Mars' planetary radius $a = 3400\,\mathrm{km}$. [8]

Discuss qualitatively how such a wave mode could be excited on Mars, and the effects its excitation might have on conditions close to the Martian surface. [5]

8. Briefly describe the main source of evidence that the early atmosphere of Mars was significantly denser than is currently observed. List three of the main processes that are likely to have led to the Martian climate seen today. [8]

Using the information given below, describe the thermal structure of the Martian atmosphere with height from the surface to an altitude of $\sim 100\,\mathrm{km}$ as predicted by a simple radiative-convective model, noting any assumptions made. Illustrate your answer by drawing a fully labelled quantitative sketch. [8]

Why, in at least one important respect, is this model more representative of Mars than the Earth? Discuss the effect of a large-scale dust storm on the atmospheric structure on Mars and illustrate this qualitatively on your sketch. [4]

Mars' early atmosphere is likely to have contained significantly more infrared absorber than in the present epoch. If the amount of absorber was increased by a factor of 5 over present amounts and the infrared absorption approximately follows the 'strong limit' (i.e. the optical depth of an atmospheric path is proportional to the square root of the number of absorbing molecules in the path), estimate the modified surface temperature, assuming that all other parameters remain constant and the scale height on Mars is 13 km. [5]

[Assume Mars' orbital distance from the Sun = 1.52 AU, Mars' Bond Albedo = 0.16, Mars' radius = 3400 km, Mars' surface temperature in this simple model = 255 K, lapse rate for a Martian CO_2 atmosphere = $4.5\,\mathrm{K\,km^{-1}}$.]