SECOND PUBLIC EXAMINATION

Honour School of Physics Part C: 4 Year Course

Honour School of Physics and Philosophy Part C

C4: PARTICLE PHYSICS

TRINITY TERM 2011

Thursday, 23 June, 9.30 am – 12.30 pm

Answer **four** *questions.*

Start the answer to each question in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do NOT turn over until told that you may do so.

Pages 2 and 3 contain particle physics formulae and data for this paper.
The questions start on page 4.

C4 Particle Physics formulae and data

Unless otherwise indicated, the questions on this paper use natural units with $\hbar = c = 1$. The energy unit is GeV.

Cross sections	$1\,\mathrm{GeV}^{-2} = 0.3894\,\mathrm{mb}$
Length	$1\,\mathrm{GeV}^{-1} = 0.1973\,\mathrm{fm}$
Time	$1\,\mathrm{GeV}^{-1} = 6.582 \times 10^{-25}\,\mathrm{s}$
Fermi constant	$G_\mathrm{F} = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$

Dirac (Dirac-Pauli representation) and Pauli matrices

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Rotation matrices $\quad \langle j, m' | \mathrm{e}^{-\mathrm{i}J_y\theta} | j, m \rangle = d^j_{m'm}(\theta)$

$$d^{1/2}_{++} = d^{1/2}_{--} = \cos(\theta/2)\,; \quad d^{1/2}_{-+} = -d^{1/2}_{+-} = \sin(\theta/2).$$

$$d^1_{11} = d^1_{-1-1} = (1+\cos\theta)/2\,; \quad d^1_{1-1} = d^1_{-11} = (1-\cos\theta)/2\,;$$

$$d^1_{00} = \cos\theta\,; \quad d^1_{01} = -d^1_{10} = -d^1_{0-1} = d^1_{-10} = \sin\theta/\sqrt{2}\,.$$

Spherical harmonics $\quad Y^m_l(\theta, \phi)$

$$Y^0_0 = \sqrt{\frac{1}{4\pi}}\,; \quad Y^0_1 = \sqrt{\frac{3}{4\pi}}\cos\theta\,; \quad Y^{\pm1}_1 = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\,\mathrm{e}^{\pm\mathrm{i}\phi}\,.$$

$$Y^0_2 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)\,; \quad Y^{\pm1}_2 = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\,\mathrm{e}^{\pm\mathrm{i}\phi}\,; \quad Y^{\pm2}_2 = \sqrt{\frac{15}{32\pi}}\sin^2\theta\,\mathrm{e}^{\pm2\mathrm{i}\phi}\,.$$

CKM quark mixing matrix

The mixing of the charge $-e/3$ quark mass eigenstates (d, s, b) is expressed in a 3×3 unitary matrix V:

$$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_\mathrm{ud} & V_\mathrm{us} & V_\mathrm{ub} \\ V_\mathrm{cd} & V_\mathrm{cs} & V_\mathrm{cb} \\ V_\mathrm{td} & V_\mathrm{ts} & V_\mathrm{tb} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}.$$

The magnitudes of the elements, derived from the Particle Data Group 2004 tables, are given below. The number in brackets gives an estimate of the uncertainty in the last digit. Note that these values may not give an exactly unitary matrix, but this has no significance.

$$V = \begin{pmatrix} 0.975(0) & 0.224(3) & 0.004(1) \\ 0.224(3) & 0.974(1) & 0.042(2) \\ 0.009(5) & 0.040(3) & 0.999(0) \end{pmatrix}$$

Clebsch–Gordan coefficients

$1 \quad \times \quad \frac{1}{2}$

m_1	m_2	J / M	$\frac{3}{2}$ / $+\frac{3}{2}$	$\frac{3}{2}$ / $+\frac{1}{2}$	$\frac{1}{2}$ / $+\frac{1}{2}$	$\frac{3}{2}$ / $-\frac{1}{2}$	$\frac{1}{2}$ / $-\frac{1}{2}$	$\frac{3}{2}$ / $-\frac{3}{2}$
$+1$	$+\frac{1}{2}$		1					
$+1$	$-\frac{1}{2}$			$\sqrt{1/3}$	$\sqrt{2/3}$			
0	$+\frac{1}{2}$			$\sqrt{2/3}$	$-\sqrt{1/3}$			
0	$-\frac{1}{2}$					$\sqrt{2/3}$	$\sqrt{1/3}$	
-1	$+\frac{1}{2}$					$\sqrt{1/3}$	$-\sqrt{2/3}$	
-1	$-\frac{1}{2}$							1

$1 \quad \times \quad 1$

m_1	m_2	J / M	2 / $+2$	2 / $+1$	1 / $+1$	2 / 0	1 / 0	0 / 0	2 / -1	1 / -1	2 / -2
$+1$	$+1$		1								
$+1$	0			$\sqrt{1/2}$	$\sqrt{1/2}$						
0	$+1$			$\sqrt{1/2}$	$-\sqrt{1/2}$						
$+1$	-1					$\sqrt{1/6}$	$\sqrt{1/2}$	$\sqrt{1/3}$			
0	0					$\sqrt{2/3}$	0	$-\sqrt{1/3}$			
-1	$+1$					$\sqrt{1/6}$	$-\sqrt{1/2}$	$\sqrt{1/3}$			
0	-1								$\sqrt{1/2}$	$\sqrt{1/2}$	
-1	0								$\sqrt{1/2}$	$-\sqrt{1/2}$	
-1	-1										1

Breit-Wigner resonance formula

The formula represents the energy dependence of the total cross-section $\sigma(i \to f)$ for unpolarised scattering between a two-body initial state i to a final state f, in the vicinity of a resonance of rest-mass energy M, spin J and total width Γ.

$$\sigma(i \to f) = \pi \lambda\!\!\!^{-2} g \frac{\Gamma_i \Gamma_f}{[(E-M)^2 + \Gamma^2/4]},$$

where $\lambda\!\!\!^{-} = \dfrac{\hbar c}{pc}$, $g = \dfrac{2J+1}{(2s_a+1)(2s_b+1)}$, p is the magnitude of the centre-of-mass momentum of the initial state particles, s_a, s_b are their spins and Γ_i, Γ_f the initial and final state partial widths.

1. Draw one Feynman diagram, illustrating the oscillation of neutral mesons such as K^0or B^0.

The physical states of the kaon system can be written as

$$\begin{aligned} |K_S\rangle &= A((1+\epsilon)|K^0\rangle + (1-\epsilon)|\overline{K}^0\rangle) \\ |K_L\rangle &= A((1+\epsilon)|K^0\rangle - (1-\epsilon)|\overline{K}^0\rangle), \end{aligned}$$

where ϵ is a small (complex) parameter. (A is a normalisation constant.) Explain the physical significance of the parameter ϵ.

Outline how the mass difference, ΔM_K, of K_S and K_L can be measured. [10]

The initial two-kaon state from the decay of a $\phi^0(1020)$ is

$$|\phi(t=0)\rangle = (|K^0(\vec{p}), \overline{K}^0(-\vec{p})\rangle - |\overline{K}^0(\vec{p}), K^0(-\vec{p})\rangle)/\sqrt{2},$$

where $|p| = 110\,\mathrm{MeV/c}$ is the laboratory momentum of the kaons. Show that the initial state can be written as a product of K_S and K_L states. [2]

In a ϕ-factory (a symmetric e^+e^- collider), K^0–$\overline{K}^0$pairs are produced coherently from the decay of $\phi^0(1020)$ mesons at rest in the laboratory. A cylindrically symmetric detector consisting of low-mass tracking detectors followed by a cylindrical calorimeter surrounds the interaction point. A magnetic field parallel to the beam direction is provided by an external solenoid. The tracking detectors extend from $r = 0.25\,\mathrm{m}$ to $r = 2\,\mathrm{m}$.

Suggest the experimental signatures by which pure samples of K_S and K_L decays can be identified with the detector. Hence explain how an unbiased sample of K_S decays can be extracted. [8]

The semi-leptonic charge asymmetry of K_L decays is defined as

$$A_L = \frac{\Gamma(K_L \to l^+\pi^-\nu) - \Gamma(K_L \to l^-\pi^+\nu)}{\Gamma(K_L \to l^+\pi^-\nu) + \Gamma(K_L \to l^-\pi^+\nu)}.$$

Why cannot A_S, the equivalent semi-leptonic charge asymmetry in K_S decays, be measured in fixed-target experiments?

A_L is measured to be $(3.32 \pm 0.06) \times 10^{-3}$. If CPT is violated, the values of the ϵ would be different for K_S and K_L. The KLOE experiment at the DAΦNE ϕ-factory measured $A_S = 1.5 \times 10^{-3}$ from a sample of 13,000 K_S decaying semi-leptonically. Discuss whether these results provide significant evidence for CPT violation. [5]

[The K^0 mass is 497.6 MeV. The mean lifetimes of K_S and K_L are 89.5 ps and 51.8 ns respectively.]

2. The four $\Delta(1232)$ states Δ^{++}, Δ^{+}, Δ^{0}, Δ^{-} are $J = \frac{3}{2}$ baryons that predominantly decay to a nucleon (N) and a pion.

Write down a simple expression relating the third component of the isospin (I_3), the baryon number (B) and the electric charge (Q) and hence identify the isospin states, $\mid I, I_3\rangle$ of each $\Delta(1232)$. Deduce the four normalised flavour wavefunctions in the $\mathrm{N} \otimes \pi$ basis. [4]

A π^+ beam is incident on a deuterium target. What momentum should the beam possess to maximise production of $\Delta(1232)$? By assuming isospin conservation, in what proportion are the following resonances produced: $\Delta^{++} : \Delta^{+} : \Delta^{0}$? Which two fundamental assumptions underpin this prediction? [6]

When considering strangeness S, in addition to I_3 the nucleons are seen to be members of the baryon, spin-$\frac{1}{2}$ octet. The $\Delta(1232)$ states are part of a decuplet. Sketch these two multiplets, remembering to label and enumerate the axes. Name every state and speculate on their quark content. An asterisk can be used to distinguish an excited state from its ground state. [4]

With reference to your diagrams, where helpful, discuss the following:

- Though the π^+ and π^- are of equal mass, the Σ^+ and Σ^- masses differ by 8 MeV/c^2.
- The lifetime of the Σ^0 is 10 orders of magnitude shorter than that of Λ^0 or Ξ^0.
- What does the observation of the Δ^{++} imply for the quark?
- What experimental signature might unambiguously identify the Ω^-?

[6]

What is U-spin? If SU(3) flavour symmetry were exact, why would the decay $\Sigma^{*-} \to \Sigma^-\gamma$ be forbidden whilst $\Sigma^{*+} \to \Sigma^+\gamma$ is allowed? [5]

[Nucleon isospin eigenstates: $\mid \mathrm{n}\rangle = \mid \frac{1}{2}, -\frac{1}{2}\rangle$; $\mid \mathrm{p}\rangle = \mid \frac{1}{2}, +\frac{1}{2}\rangle$
Pion isospin eigenstates: $\mid \pi^+\rangle = \mid 1, +1\rangle$; $\mid \pi^0\rangle = \mid 1, 0\rangle$; $\mid \pi^-\rangle = \mid 1, -1\rangle$
$S = -1, -2, -3$ baryons can be labelled Σ (except the Λ^0), Ξ, Ω respectively.
Particle masses in MeV/c^2: n, 940; p, 938; $\pi^\pm, 140$; $\pi^0, 135$.]

3. For each of the following processes, give an argument (quantitative, where possible) whether the final state given is the most likely final state and in cases where it is clearly not, give an example of a more likely one. If it is strongly suppressed, say why. Detailed phase space calculations are not required. For the W, t and H consider the decays into quarks and ignore the step where hadrons are formed. All the processes are energetically allowed.

(a) $\Delta^0 \to \mathrm{p}\,\pi^-$

(b) $\mathrm{K}^0_\mathrm{S} \to \mathrm{K}^-\mathrm{e}^+\nu_e$

(c) $\mathrm{H} \to \mathrm{ZZ}$ for a Higgs mass of 175 GeV/c^2

(d) $\mathrm{D}^0 \to \mathrm{K}^+\,\pi^-$

(e) $\mathrm{t} \to \mathrm{Wb}$

(f) $\mathrm{B}_c^+ \to \mathrm{B}_s e^+\nu_e$, where B_c^+ is a meson containing b and c quarks

(g) $\mathrm{K}^0_\mathrm{L} \to \mu^+\mu^-$

(h) $\mathrm{W}^+ \to \mathrm{e}^+\nu_e$

(i) $\pi^- \to \mathrm{e}^-\,\overline{\nu}_e$ [25]

4. The mass and width of the W boson can be measured at LEP and at the Tevatron. Draw the leading order Feynman diagrams leading to W production (a) in $\mathrm{e}^+\mathrm{e}^-$ collisions and (b) in $\overline{\mathrm{p}}\mathrm{p}$ collisions. Explain why the μ^+ from $\mathrm{W}^+ \to \mu^+\nu_\mu$ in $\overline{\mathrm{p}}\mathrm{p}$ collisions are more likely to emerge with a momentum component parallel to the anti-proton momentum rather than anti-parallel to it. [7]

Discuss how the cross section for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^-$ varies as a function of $\sqrt{s}$ in the region around $\sqrt{s} \sim 160$ GeV and how this can be used to determine the mass of the W. In a particular event of this type, two jets are reconstructed with momenta of $(p_x, p_y, p_z) = (-12.31,\ 13.35,\ -40.79)$ GeV/c and $(-12.92,\ 26.04,\ 35.18)$ GeV/c, along with an electron with momentum $(16.52,\ -63.65,\ 12.72)$ GeV/c. Obtain estimates of the W mass from this event. [8]

Estimate the fraction of $\mathrm{Z} \to \mathrm{WW}$ events which appear as

(a) four jets,

(b) two jets and an electron,

(c) two jets and a muon,

(d) any other topology. [5]

Discuss whether it is plausible to measure the width of the W and/or the width of the Z using $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW}$ events. [5]

$[\Gamma_W \sim 2.1\,\mathrm{GeV},\ \Gamma_\mathrm{Z} \sim 2.5\,\mathrm{GeV}]$

5. Two positive-energy solutions of the Dirac equation are

$$\psi_s(\mathbf{r},t) = N \begin{pmatrix} \boldsymbol{\chi}^s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\boldsymbol{\chi}^s \end{pmatrix} \mathrm{e}^{\mathrm{i}(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar},$$

$$\text{where} \quad s = 1, 2\,. \qquad \boldsymbol{\chi}^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\chi}^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Determine the normalization factor N and explain why these wave functions are not normalized to one particle per unit volume. Which quantum numbers uniquely identify a free electron state? [7]

Time inversion is achieved through complex conjugation of the wavefunction, followed by the application of the $\mathrm{i}\gamma^1\gamma^3$ operator. Show that under time inversion, the magnetic dipole moment of a free electron changes its direction. Describe what is meant by Kramers degeneracy and prove it. [8]

Briefly describe why the neutron's electric dipole moment is expected to be negligible and why it is interesting to measure it with high precision. How could one measure it? [10]

6. Write short accounts of **three** of the following:

(a) Neutrinoless double beta decay.

(b) Solar neutrino detection.

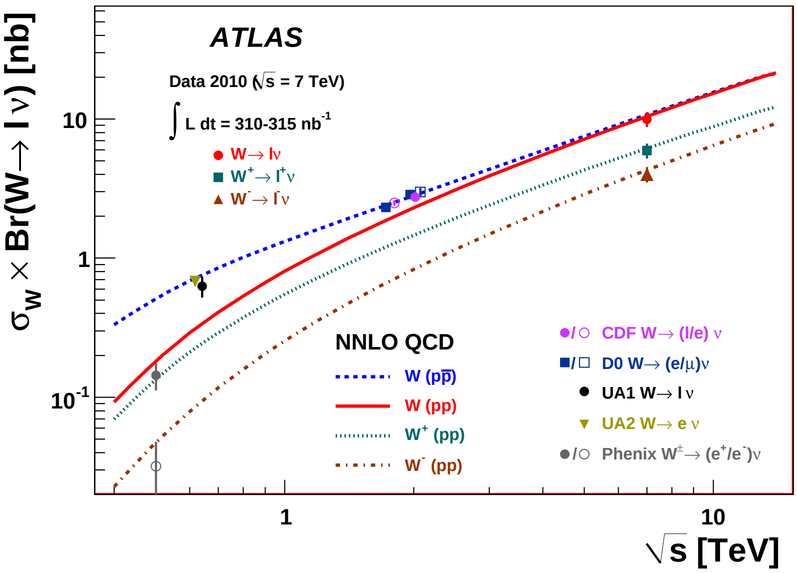
(c) Experimental search for particle dark matter.

(d) The physical principles of silicon micro-strip detectors and their role in high-energy spectrometers.

(e) The limitations of the Standard Model of Particle Physics. [25]

7. Draw example Feynman diagrams of (a) W decay to leptons and (b) W decay to quarks. Estimate the branching ratio $BR(\mathrm{W}^+ \to \mathrm{e}^+\nu)$. [4]

In a hadron-hadron collision, partons carrying momentum fractions x_a and x_b collide. Find an expression for the centre of mass (cms) energy of this parton-parton collision in terms of the cms energy of the hadron-hadron collision $\sqrt{s}$. Evaluate typical values of parton x for W production at the Tevatron ($\sqrt{s} = 2\,\mathrm{TeV}$) and LHC ($\sqrt{s} = 7\,\mathrm{TeV}$).



The figure shows measured and calculated values of the cross section times branching ratio for $\mathrm{W}^\pm \to e^\pm\nu$ in $\overline{\mathrm{p}}\mathrm{p}$ and pp collisions.

Explain why the $\overline{\mathrm{p}}\mathrm{p}$ cross section for W production is very much larger than for pp at Tevatron energies but the difference between the two becomes negligible at high energies. [9]

Give a brief description of a suitable detector for measuring the energies of electrons from W decays at the LHC, explaining the detection technique used. [3]

Give an example of a background source that could mimic high transverse momentum electrons and explain how high transverse momentum electrons could be identified in the presence of such a background.

How is it possible to measure the transverse momentum of a neutrino in an event in a hadron-hadron collision? Why is the measurement of high energy but low transverse momentum neutrinos much more difficult? [9]

8. The net potential for non-relativistic elastic scattering of an electron from a static spherical distribution of charges carrying Yukawa potentials is

$$V(\mathbf{r}) = g^2 \int \mathrm{d}^3\mathbf{r}'\, \frac{\rho(\mathbf{r}')\mathrm{e}^{-\mu R}}{R},$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, $R = |\mathbf{R}|$ and $\int \mathrm{d}^3\mathbf{r}'\, \rho(\mathbf{r}') = 1$. Given that $F(q^2) = \int \mathrm{d}^3\mathbf{r}'\, \rho(\mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}'}$ is the *form factor* of the charge distribution, use the Born approximation to show that the differential cross-section may be written

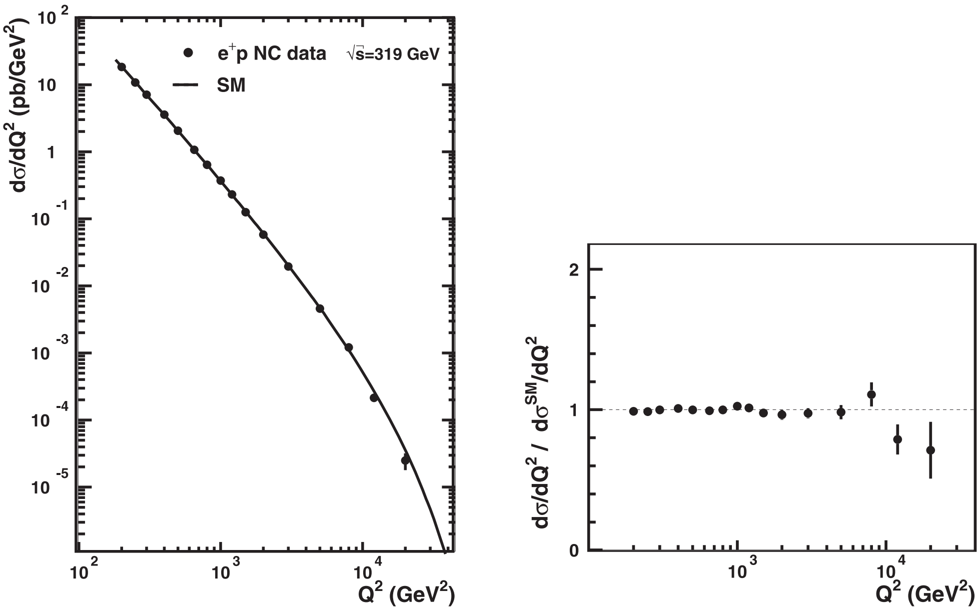
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right]_{\mathrm{point}} \times |F(q^2)|^2,$$

where $[\mathrm{d}\sigma/\mathrm{d}\Omega]_{\mathrm{point}}$ is the cross-section for pointlike Yukawa scattering and $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ is the difference between the 3-momenta $\mathbf{k}$ and $\mathbf{k}'$ of the incident and scattered electron. [10]

The charges are effectively confined within a sphere of radius r'_{max}. Show that for qr'_{max} small the form factor may be approximated by

$$F(q^2) = 1 - aq^2\,\langle r^2\rangle,$$

where $\langle r^2\rangle$ is the mean-square radius of the charge distribution. Determine the value of a. [8]



The figure shows the differential cross-section $\mathrm{d}\sigma/\mathrm{d}Q^2$ for neutral current scattering $\mathrm{e}^+\mathrm{p} \to \mathrm{e}^+X$ at a centre-of-mass energy of 319 GeV as measured by the H1 experiment at the HERA collider. The data are in good agreement with the standard-model calculation in which both positrons and quarks are taken to be point-like objects. Using the above form-factor approach, estimate an upper limit for the rms radius of the electroweak charge distribution of a quark. [7]

[In the Born approximation one assumes that the wave-functions of the incident and scattered particles may be represented by plane waves outside the range of the potential.]