

SECOND PUBLIC EXAMINATION

Honour School of Physics Part C: 4 Year Course

Honour School of Physics and Philosophy Part C

C1: ASTROPHYSICS

TRINITY TERM 2011

Saturday, 25 June, 9.30 am – 12.30 pm

*Answer **four** questions.*

Start the answer to each question in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do NOT turn over until told that you may do so.

1. Consider a universe consisting of cold dark matter (CDM) and an extra component that has negligible clustering on small scales such as radiation or a cosmological constant. The cold dark matter density perturbation contrast (δ) equation, on small scales, can be expressed as

$$x^2 (1 + x^{-3w}) \frac{d^2 \delta}{dx^2} + \frac{3}{2} x (1 + (1 - w)x^{-3w}) \frac{d\delta}{dx} - \frac{3}{2} \delta = 0$$

where w is the equation of state of the extra component, $x = a/a_{\text{eq}}$ and a_{eq} is the scale factor when the cold dark matter density equals the other component's density. Once one solution (δ_1) to this equation is found the second (δ_2) can be found using the properties of the Wronskian:

$$\delta_2(x) = \delta_1(x) \int^x \frac{dy}{y^{3/2} (1 + y^{-3w})^{1/2} \delta_1(y)^2}.$$

Find the early time solution to δ when the extra component is a cosmological constant ($w = -1$). Show that the late time behaviour is δ approximately constant. Give the underlying physical reasons for these solutions. [10]

Now consider the case where the extra component is radiation. Find the late time solution. Show that the early time solution is $\delta_1 \approx C_1 + C_2 \log(x)$ where C_1 and C_2 are constants. Explain the underlying physics behind these solutions. [10]

Now consider the case where the universe consists of CDM, a cosmological constant, and radiation. Use the solutions to the earlier part of this question to sketch the behaviour of δ as the Universe goes from radiation domination to matter domination to cosmological constant domination. [5]

2. All density-wave models of spiral structure in galaxies require that:

- galactic discs are quite stable, otherwise there would be very few disc galaxies;
- but also that galactic discs are sufficiently unstable, otherwise the density waves would be damped out very quickly.

The trade-off between these two conditions is encapsulated in a parameter Q , called the Toomre parameter. The aim of this question is to derive an expression for this parameter in terms of the properties of the disc.

First, identify the forces which stabilise and destabilise the disc. [2]

Now suppose there is a stationary star at the edge of a spherical region of radius R and mass M . Using Kepler's third law ($P^2 = 4\pi^2 a^3 / (MG)$ where P is the orbital period of the test particle, a its semi-major axis and G is Newton's gravitational constant), estimate the time t_{coll} it will take the particle to fall to the centre of the region. [3]

Strictly speaking the previous calculation is only valid for a spherically symmetric system, but it yields a reasonable approximation for discs. Assuming the disc is an infinitely thin cylinder of radius R with uniform surface density Σ , rewrite the previous expression for t_{coll} in terms of Σ . [3]

In the absence of gravity, estimate t_{esc} , the time it would take a star moving at velocity $\sigma = \sqrt{\langle v^2 \rangle}$ to escape the region. What is the critical radius, also called Jeans length, L_J , beyond which a star is trapped in the collapse? Hence state the condition for a disc to be stable against gravitational collapse. [4]

Suppose that the edge of a disc of radius R_0 is rotating with angular velocity ω . What is the angular momentum per unit mass at the edge of the disc? If the disc contracts to a new radius R , what amount of centrifugal acceleration does it undergo? Assuming that the disc is an infinitely thin cylinder of radius R_0 with uniform surface density Σ , calculate the gravitational force exerted on a unit mass when the disc contracts to radius R , and derive the critical radius, L_{rot} beyond which the disc is stable to an infinitesimal contraction dR . [6]

Combine the two stability criteria derived above to deduce an expression for the critical velocity dispersion σ_{crit} and therefore the stability parameter of the disc $Q \equiv \sigma / \sigma_{\text{crit}}$ in terms of the properties of the disc (Σ , ω) and the fundamental constants of the problem. [4]

What is the range of Q values for which the disc is stable? What do you think happens to a spiral density wave perturbation when Q approaches the critical value from the stable side? How does the disc respond to the perturbation? [3]

3. Consider a black hole of mass M moving at speed v through a cold medium of density ρ . By simple scaling arguments show that material will accrete onto the black hole if it lies within the *Bondi-Hoyle radius*,

$$R_{\text{BH}} \sim \frac{2GM}{v^2},$$

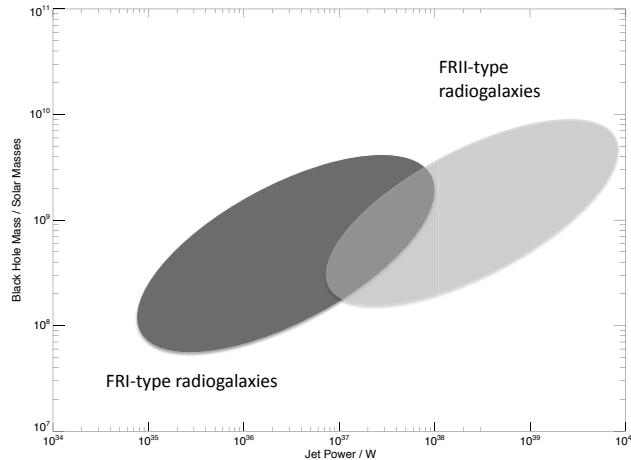
with an accretion rate of approximately

$$\dot{M} \sim \pi R_{\text{BH}}^2 \rho v. \quad [4]$$

Now consider a stationary black hole in a hot medium of density ρ and sound speed c_s . In such a situation it may be argued that accretion will occur from within a spherical region where the sound speed is less than the velocity of escape from the black hole. By analogy with the calculation above, show that the accretion rate is now given by

$$\dot{M} \sim \frac{16\pi\rho G^2 M^2}{c_s^3}. \quad [3]$$

In the central region of a radio-loud AGN, the gas near the black hole typically has a temperature of ~ 1 keV and a particle density of $\sim 10^5 \text{ m}^{-3}$. Assuming that the AGN is powered by accretion onto the black hole in a manner similar to that described above, derive a relationship between the power of the radio jet and the mass of the black hole. Clearly state any assumptions made. [10]



The figure, adapted from Hardcastle et al. (2007), shows the outlines of the regions occupied by FRI and FRII radiogalaxies in the black hole mass–radio jet power plane. Discuss how well the model developed above fits the observed distribution. [5]

Does your analysis suggest any explanation for the fact that, in the nearby Universe, FRI radiogalaxies tend to lie in galaxy clusters, whereas FRII radiogalaxies tend to lie in the field? What observational evidence might you look for to test such an explanation? [3]

4. In a radiosource, a relativistic electron with a Lorentz factor γ passes through a region of uniform magnetic flux density of magnitude B . Assuming the electron orbit is perpendicular to the magnetic field, show that the electron has an orbital frequency of

$$\nu = \frac{eB}{2\pi\gamma m_e}.$$

Describe the process of emission of synchrotron radiation from the electron, in particular showing that the spectrum of energy radiated by the electron is strongly peaked at a characteristic frequency

$$\nu_{\text{crit}} \sim \frac{\gamma^2 eB}{2\pi m_e}. \quad [8]$$

The total synchrotron power radiated by the electron is given by

$$P = \frac{2\gamma^2 \beta^2 c \sigma_T B^2}{3\mu_0},$$

where $\beta = \frac{v}{c}$ and σ_T is the Thomson cross section. Outline how this may be used to infer the “spectral age” of a population of synchrotron-emitting electrons, describing how the relevant physical quantities are estimated.

The nearby radiogalaxy Centaurus A has been well-studied at low radio frequencies and shows a break in its radio spectrum at a frequency of about 1.0 GHz. Taking the magnetic flux density in the lobes to be about 0.3 nT, estimate the time since the lobe electrons were last accelerated to relativistic energies. [7]

Now consider an inverse-Compton collision between a relativistic electron of Lorentz factor γ and a very low-energy photon. By consideration of the expression for total synchrotron power given above, or otherwise, show that the photon recoils with an energy roughly γ^2 times greater than its initial energy. [5]

In 2010 a team of astromomers using the *Fermi* Gamma-Ray Space Telescope announced they had discovered a significant flux of very high-energy photons, up to 100 GeV, from the lobes of Centaurus A. What are the implications for the simple “spectral aging” picture outlined above? What mechanisms may be involved in the creation of such high-energy photons? [5]

5. Starting with a binary system on the main sequence (MS), sketch the various evolutionary phases that lead to the formation of a *millisecond pulsar*. [8]

Show that the total angular momentum of a binary, consisting of two stars of masses M_1 and M_2 , can be written as

$$J = \mu \sqrt{GM_T A},$$

where A is the orbital separation, μ the reduced mass of the system, and $M_T = M_1 + M_2$. [3]

Consider a low-mass X-ray binary, consisting initially of a $1.4 M_\odot$ neutron star and a $1.7 M_\odot$ main-sequence companion. Assuming that mass transfer is conservative, determine the evolution of A as a function of the secondary mass M_2 . Sketch both A as a function of M_2 and M_2 as a function of time t since the beginning of mass transfer, estimating the properties of the system (i.e. the orbital separation and the component masses) at the beginning and at the end of the mass-transfer phase. Clearly indicate phases of ‘fast’ and ‘slow’ mass transfer and estimate their durations.

[MS mass–radius relation: $R \simeq R_\odot (M/M_\odot)^{2/3}$; MS lifetime: $T_{\text{MS}} \simeq 10^{10} (M/M_\odot)^{-3} \text{ yr}$; Kelvin-Helmholtz timescale: $t_{\text{KH}} \simeq GM^2/2RL$; Roche-lobe radius (star 2): $R_{\text{RL}} \simeq 0.4 A (M_2/M_1)^{1/3}$; radius of a (sub-)giant as a function of degenerate core mass m_c : $R_g \simeq 10^3 R_\odot (m_c/M_\odot)^3$.] [10]

A binary millisecond pulsar is found in a very eccentric orbit ($e = 0.5$, where e is the eccentricity) with a $0.2 M_\odot$ white dwarf and a semi-major axis of $3 R_\odot$. Explain what makes this system very unusual. Speculate on the evolutionary history of this system if it is (a) found in a globular cluster and (b) in the Galactic disc. [4]

6. Briefly describe how a protostar forms in a molecular-cloud core that is only supported by gravity. Explain the importance of the *Jeans mass* M_J and the possible role of rotation. [8]

The Jeans mass can be estimated as

$$M_J \simeq 6 M_\odot \left(\frac{T}{10 \text{ K}} \right)^{3/2} \left(\frac{n_{\text{H}_2}}{10^{10} \text{ m}^{-3}} \right)^{-1/2},$$

where T is the temperature of the core and n_{H_2} the number density of molecular hydrogen.

Use this relation to estimate the mass density and the characteristic size of a $1 M_\odot$ marginally Jeans-stable isothermal core with $T = 10 \text{ K}$. Estimate the characteristic timescale on which the core collapses. Using this timescale, or otherwise, estimate the accretion rate onto the central protostar during the collapse phase. [6]

The velocity gradient in a typical core across the core is observed to be $dv/dR = 1 \text{ km s}^{-1} \text{ pc}^{-1}$. Use this to estimate the characteristic size of any accretion disc that may form around the protostar. Comment on its relevance for the formation of planetary systems. [6]

Consider a star with a planetary system at the centre of a globular cluster with a high stellar density $n \simeq 10^5 \text{ pc}^{-3}$ and a typical velocity dispersion $v \simeq 10 \text{ km s}^{-1}$. Estimate the maximum separation at which a planet can survive for 10^{10} yr . [Hint: consider the two-body collision time $t_{\text{coll}} = 1/n\sigma v$ and determine the critical collision cross section σ]. [5]

7. Describe the processes and quantities that determine the radius of ionized hydrogen surrounding a hot (O-type) star in the Galaxy. [5]

Estimate the radii of the HII and HeII regions formed from material with a hydrogen density $n(\text{H}) = 10^{10} \text{ m}^{-3}$ surrounding stars with $T_{\text{eff}} = 32,000 \text{ K}$ and $40,000 \text{ K}$. The recombination coefficients and numbers of ionizing photons are given below. Discuss any assumptions or approximations made in these estimates. [10]

Describe the spectrum of light that emerges from the HII regions. If the surrounding material had a density 3 times lower, what would the effect on the visibility of the HII regions be? [5]

Small dust grains of radius 0.5 nm exist immediately outside the HII regions. Describe the temperature history of these particles, providing estimates of relevant timescales. [5]

	Photon fluxes (s^{-1})	
	$N(53 < \lambda < 91 \text{ nm})$	$N(\lambda < 53 \text{ nm})$
$T_{\text{eff}} = 40,000 \text{ K}$	8.3×10^{48}	1.3×10^{48}
$T_{\text{eff}} = 32,000 \text{ K}$	5.1×10^{47}	2.3×10^{45}

[The recombination coefficients are: $\alpha(\text{H}) = 2.6 \times 10^{-19} \text{ m}^3 \text{ s}^{-1}$,
 $\alpha(\text{He}) = 4.1 \times 10^{-19} \text{ m}^3 \text{ s}^{-1}$]

8. Write an essay on *one* of the following topics. Your essay should include qualitative discussion of the essential physical principles and astronomical observations as well as mathematical descriptions of the physical laws and relationships where relevant.

- The Cosmic Microwave Background (CMB) provides a gold-mine of information about the Universe. Describe the basics physics behind the CMB and how CMB observations are analyzed.
- Describe what is meant by a gravitational lens and discuss the different classes of gravitational lensing.

[25]