XPHA 4279

SECOND PUBLIC EXAMINATION

Honour School of Physics and Philosophy Part A

A2P: ELECTROMAGNETISM

Tuesday 14 June 20011, 2.30 pm – 4.10 pm

Time allowed: 1 hour 40 minutes

Answer **all** *of Section A and* **two** *questions from Section B.*

Start the answer to each question on a fresh page.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

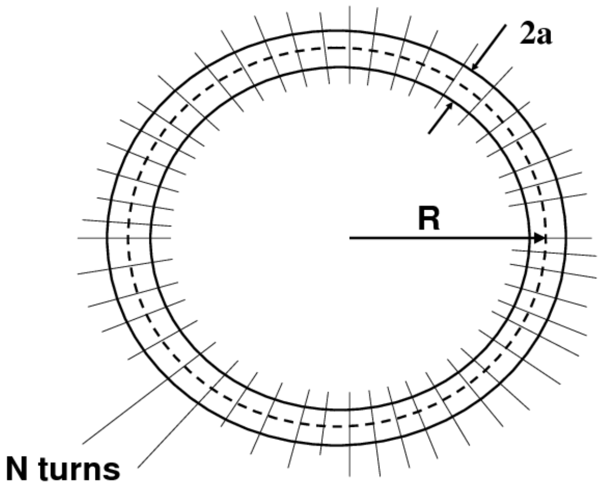
1. Consider two thin long coaxial conducting cylinders of radii a and b with $b > a$. The potential difference between the two cylinders is a constant V. Determine the electric field in the region between the two cylinders. Evaluate the capacitance per unit length. [6]

A capacitor is made from such an arrangement with $a = 10$ mm and $b = 20$ mm. What length should be used to produce a capacitance of 10 pF? Neglect edge effects. [2]

2. Briefly explain the laws of electromagnetic induction of Faraday and Lenz. [2]

A planar coil of radius R is placed in a region of uniform time-dependent magnetic flux density $\mathbf{B} = \mathbf{B_0} \sin \omega t$. The angle between the $\mathbf{B}$ field and the coil's normal is θ. Find an expression for the resulting electromotive force (emf) in the coil. [4]

3.



A toroid has a mean radius R and an inner radius a. It has N turns of thin wire wrapped uniformly around the toroid as shown in the figure, carrying a current I. Find an expression for the magnetic flux density in the toroid and hence determine the self-inductance of the circuit. For such a toroid with $N = 1000$, $R = 1$ m, $a = 5$ mm, and $I = 20$ A, evaluate the energy stored in the magnetic field. [8]

4. Write down Maxwell's equations in a vacuum in differential form. Show that they allow wave solutions for the electric field $\mathbf{E}$ and the magnetic flux density $\mathbf{B}$, and show that the polarisation of $\mathbf{E}$ is perpendicular to $\mathbf{B}$ for such a wave. [5]

Section B

5. A point Q is located at a distance r from an electric dipole of magnitude p. The angle between the dipole and the line joining the dipole to the point Q is θ. Calculate the potential at the point Q assuming r is large. [8]

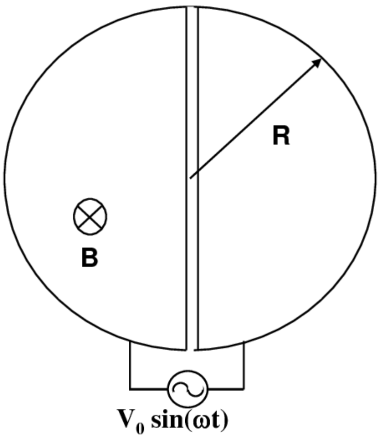
Explain briefly the method of image charges in electrostatics. [2]

A dipole is placed at a coordinate (0,0,d) above an infinite conducting grounded sheet which lies in the (x, y) plane. The dipole is oriented with its axis pointing in the $\hat{\mathbf{z}}$ direction. Determine an expression for the potential in the region above the grounded plane. Hence find an expression for the z component of the electric field $\mathbf{E}_z$ as a function of x and y for $0 < z \ll d$. [6]

What is the surface charge density on the sheet at the point $(x, y) = (0, 0)$? [4]

6. State the Lorentz force law applied to a particle of charge q moving with a velocity $\mathbf{v}$ in a region of space with electric field $\mathbf{E}$ and magnetic flux density $\mathbf{B}$. If $\mathbf{E} = 0$ and $\mathbf{B}$ is time independent, explain whether it is possible to increase the kinetic energy of the charged particle. [4]

Show that a particle of charge q in a region of uniform magnetic flux density $\mathbf{B}$ and $\mathbf{E} = 0$ can undergo circular motion and determine the angular frequency ω. [4]



In a cyclotron, there exists a region of uniform magnetic flux density $\mathbf{B}$ within a thin disk of radius R. An alternating voltage $V = V_0 \sin(\omega t)$ is applied between the two halves of the cyclotron as shown in the figure. A proton is placed in the centre. What is the maximum value of the kinetic energy T_{max} of the proton that could be achieved in the cyclotron. Evaluate T_{max} for the case of protons in a cyclotron with $R = 1$ m and $B = 0.5$ T and hence calculate the maximum speed of the protons. [You may assume that the motion is always non-relativistic]. [10]

What angular frequency ω should be used to accelerate the proton? [2]

7. State the Biot-Savart law which describes the magnetic flux density $\mathbf{B}$ at a position $\mathbf{r}$ from a current element $I\mathbf{dl}$.

Evaluate the magnetic flux density $\mathbf{B}$ at a perpendicular distance y from the centre of a straight wire of length a carrying a current I. Determine an expression for $\mathbf{B}$ if $y \ll a$ and compare the result with what would be expected from an application of Ampère's law. [10]

A planar square loop of wire, with sides of length a carries a current I. Evaluate the magnetic flux density $\mathbf{B}$ at a perpendicular distance y away from the centre of one of the sides of the loop, in the plane of the loop. Find an expression for $\mathbf{B}$ if $y \gg a$ and comment on the physical significance of your result. [10]